Math 323 - Formal Mathematical Reasoning and Writing

Problem Session

Wednesday, 4/1/15

1. Which of the following is equivalent to the statement $x \notin A \cup B$?
   (a) $x \in A \cap B$
   (b) $x \notin A$ and $x \notin B$
   (c) $x \notin A$ or $x \notin B$.
2. Which of the following are equivalent to the statement $x \in \bigcap_{i \in I}(A \cup B_i)$?
   (a) For every $i \in I, x \in A \cup B_i$.
   (b) Either $x \in A$, or $x \in B_i$ for all $i \in I$.
   (c) Either $x \in A$ for every $i \in I$, or there exists an i such that $x \in B_i$.
   (d) For each $i \in I$, either $x \in A$ or $x \in B_i$.
   (e) For some $i \in I$, x is in both A and B_i.
3. True or false.
   (a) $\{\{\emptyset\}\} \cup \emptyset = \{\emptyset, \{\emptyset\}\}$
   (b) $\{\{\emptyset\}\} \cup \{\emptyset\} = \{\emptyset, \{\emptyset\}\}$
4. [1] Prove that $\displaystyle\bigcap_{n \in \mathbb{N}} \left(0, \frac{n+1}{n}\right) = (0, 1]$
5. Recall that a set S of real numbers is *convex* if the following property holds: If a and b are in S and c is a real number such that $a < c < b$, then c is also in S.

   Here are some preliminary questions to help you think about convex sets (these are the first questions I asked myself to try to get a handle on the definition).
   (a) Is the set with no elements a convex set?
   (b) Can a convex set have exactly one element?
   (c) Can a convex set have exactly two elements?
   (d) Suppose A is a convex set, and $a, b \in A$. What else can I say about A?
   (e) Can you classify convex sets of real numbers in some way? (I mean, could you list different 'types' of convex sets of real numbers?)
   (f) What do you think you should prove about the set A in the problem?

[1] Madden §9.2 #9

LaTeX tip of the week!

Here are unions and intersections in LaTeX:

`$A \cap B$`	$A \cap B$
`$A \cup B$`	$A \cup B$
`$\bigcap_{i \in I} A_i$`	$\bigcap_{i \in I} A_i$
`$\bigcup_{i \in I} A_i$`	$\bigcup_{i \in I} A_i$
`$\displaystyle \bigcap_{i \in I} A_i$`	$\displaystyle \bigcap_{i \in I} A_i$
`$\displaystyle \bigcup_{i \in I} A_i$`	$\displaystyle \bigcup_{i \in I} A_i$

Also, don't forget about `\left` and `\right`:

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\[ A \setminus ( \bigcup_{n \in \NN} \frac{1}{n} ) \]
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$$A \setminus (\bigcup_{n \in \mathbb{N}} \frac{1}{n})$$

is not as good as:

```
\[ A \setminus \left( \bigcup_{n \in \NN} \frac{1}{n} \right) \]
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$$A \setminus \left(\bigcup_{n \in \mathbb{N}} \frac{1}{n} \right)$$